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# ESTIMATE FUZZY RELIABILITY FUNCTION OF THE TRANSFORMED KAPPA MODEL WITH PRACTICAL APPLICATION

Maryam S. Kadhim Al-Naqqash1, a),
S.F. Mohamaad1
and Shrook A.S AL-Sabbah1
1Statistics Department, College of Administration and Economics,
University of Kerbaka, Iraq
a) Corresponding author: maryam.sadiq@s.uokerbala.edu.iq

**Abstract**
In this research, we conduct an experimental statistical analysis using simulation and real-life application on unusual life data characterized by fuzziness. Specifically, we study the downtime of towers belonging to Zain Iraq Telecommunications Company, which are sometimes not recorded when they are repaired. We successfully handle and analyze this fuzzy data by proposing a distribution called "Alpha Power kappa" distribution. Which was built using a transformation formula called (Alpha Power Transformed). and derive the fundamental properties of the distribution, we estimate the parameters and reliability function using estimation methods such as maximum likelihood, Kremer-von Mises method, and method of moments. By comparing, the results obtained from the simulation experiment, using statistical criteria such as mean squared error (MSE), and integrated mean squared error (IMSE). We compare the performance of these estimation methods. We find that the maximum likelihood method is the best among these methods for estimation. Furthermore, by relying on statistical criteria such as Akaike information criterion (AIC), corrected Akaike information criterion (AICc), and Bayesian information criterion (BIC). We conclude that the proposed Alpha Power kappa distribution outperforms the original kappa distribution in representing real data.

**Keywords**: kappa distribution, Alpha Power Transformed, Alpha Power Transformed kappa, simulation Maximum likelihood estimation (MLE).

## Introduction

Fuzzy sets have been a topic of interest among scientists and researchers due to the imprecision often associated with measuring complex phenomena in the real world. Owing to the inherent vagueness of human concepts and limitations of current systems, it is difficult to obtain precise measurements or deal with definitively with certain concepts. The use of fuzzy sets has proven particularly useful in addressing such challenges, offering solutions for numerous real-world problems. However, the nature of data is of utmost importance to researchers, and obtaining directly suited data for statistical methods that provide accurate estimates remains rare. Any breaches in the required conditions or inaccuracies in the measurements must be addressed with alternative methods. To this end, the study of fuzzy data constitutes an essential component to ascertain the reliability function of the distribution (alpha power kappa), using estimation methods such as the maximum likelihood method, the Kremer-von Mises method, and the method of partial estimators for data consisting of fuzzy numbers. Therefore, fuzzy sets present a promising area of research for furnishing solutions for imprecision in measuring complex phenomena.

Certainly. The study of fuzzy sets and their application to dealing with imprecision in measurements has become increasingly important in recent years as it provides a powerful tool for dealing with the inherent vagueness of human concepts and limitations often associated with current measurement systems. Fuzzy sets offer an alternative approach by considering the degrees of membership of elements to a set instead of the traditional binary approach where an element either belongs to a set or does not. One of the main applications of fuzzy sets has been in the field of control theory, which deals with the regulation of continuous variables such as temperature, pressure, and velocity. Fuzzy sets have been used to develop fuzzy controllers, which can control complex systems that are difficult to model analytically.

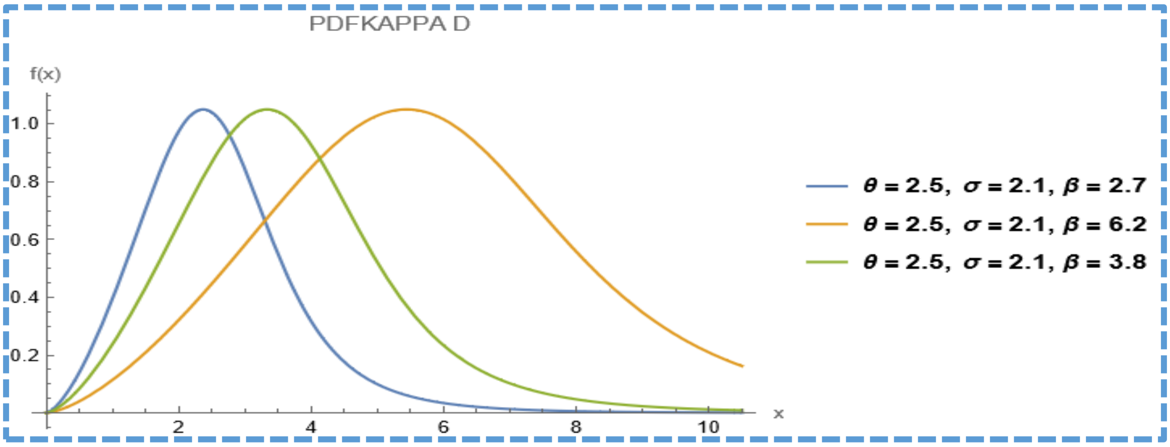
Furthermore, due to the increasing importance of artificial intelligence and machine learning, fuzzy set theory has also been applied to areas such as pattern recognition, image processing, and decision-making processes. Fuzzy set theory offers an alternative approach to dealing with uncertainty and incomplete information, which is an inherent part of these fields. Effective use of fuzzy sets requires the development of appropriate membership functions and inference mechanisms. Membership functions are used to define the degree of membership of an element to a set, while inference mechanisms are used to reason about fuzzy sets and make decisions based on fuzzy inputs.

In conclusion, the study of fuzzy sets is of great importance in dealing with imprecision and uncertainty in measurements, control theory, artificial intelligence, and machine learning. Developing appropriate membership functions and inference mechanisms is essential for effectively using fuzzy sets in practical applications.

## Kappa Distribution

The Kappa distribution is a continuous probability distribution that is extensively employed to investigate the stochastic behavior of scientifically and practically significant phenomena. Notably, it has undergone noteworthy advancements pioneered by Hosking (1994) and subsequent researchers. These advancements encompass the development of robust parameter estimation techniques by Hutson (1998), Ani Shabri and Abdul Aziz (2010), Samir (2011), and Dhwyia Hassan, Inam Abdulrahman, and Layla Nassir. The researchers concluded that the optimal approach for studying the target phenomenon involves employing the estimation method that yields superior parameter estimates for the distribution model. Furthermore, certain researchers have extended the distribution formulation from two parameters to encompass three and even four parameters, specifically focusing on studying the characteristics of the outer space domain. Consequently, this distribution finds extensive application in elucidating various aspects of outer space phenomena and atmospheric dynamics, including particle velocity, properties, space plasma, plasma temperature, as well as natural and biological manifestations such as storms, rainfall, and floods. It is noteworthy to mention that the distribution formulation in this study is a composite model comprising the Gamma distribution and the Log-normal distribution. This formulation, initially proposed by Earl S. Johnson Jr. and Paul W. Milke in their seminal work published in 1964, provides an apt representation of the Kappa distribution's probability density function. Regrettably, the specific equation representing the probability density function of the Kappa distribution is not provided within the current context.

$$f(x,\sigma,\theta,\beta)=\begin{cases}\frac{\sigma\theta}{\beta}\left(\frac{x}{\beta}\right)^{\theta-1}\left[\sigma+\left(\frac{x}{\beta}\right)^{\theta\sigma}\right]^{-\left(\frac{\sigma+1}{\sigma}\right)} & , \quad (\text{if } x>0;\ \sigma,\theta,\beta>0\ ) \\ 0 & , \quad \text{otherwise}\end{cases} \quad \dots (1)$$




The illustrative diagram in Figure (1-2) shows graphically the behavior of the probability density function of the
kappa distribution:

Figure 1 Behavior of the probability density function of the Alpha Power Transformed kappa distribution.

Alpha Power Transformed:

If the base distribution has a cdf distribution function, then the random variable (x) can be obtained from the transformed distribution by ((APT), where the aggregate density function and the probability density function (APT k distribution) can be obtained through the following formulas:

$$F(x)_{APT} = \begin{cases}\frac{\alpha^{F(x)} - 1}{\alpha - 1} & \text{if } \alpha > 0, \alpha \neq 1, x\epsilon R \\ F(x) & \text{if } \alpha > 0, \alpha = 1, x\epsilon R\end{cases} \quad \dots (4)$$

Where: α: represents the conversion parameter.

$$f(x)_{APT} = \begin{cases} \left(\frac{\log\alpha}{\alpha-1}\right) f(y)\ \alpha^{F(y)} & \text{if } \alpha > 0, \alpha \neq 0, y\epsilon R \\ f(y) & \text{if } \alpha > 0, \alpha = 0, y\epsilon R \end{cases} \quad \dots (5)$$

The reliability function of a distribution of Alpha Power Transformed kappa
Referring to equation (2-3), the survival function of the (APTk) distribution can be formulated as follows:

$$R(x,\sigma,\theta\beta,\alpha)_{APTK} = 1 - F(x,\sigma,\theta\beta,\alpha)_{APTK} = 1 - \frac{\alpha^{\left[\frac{\left(\frac{x}{\beta}\right)^{\theta\sigma}}{\sigma+\left(\frac{x}{\beta}\right)^{\theta\sigma}}\right]^{\left(\frac{1}{\sigma}\right)}} - 1}{\alpha - 1} \quad \dots (8)$$

The illustrative diagram in Figure (2-7) shows graphically the behavior of the survival function of the A P T k distribution:

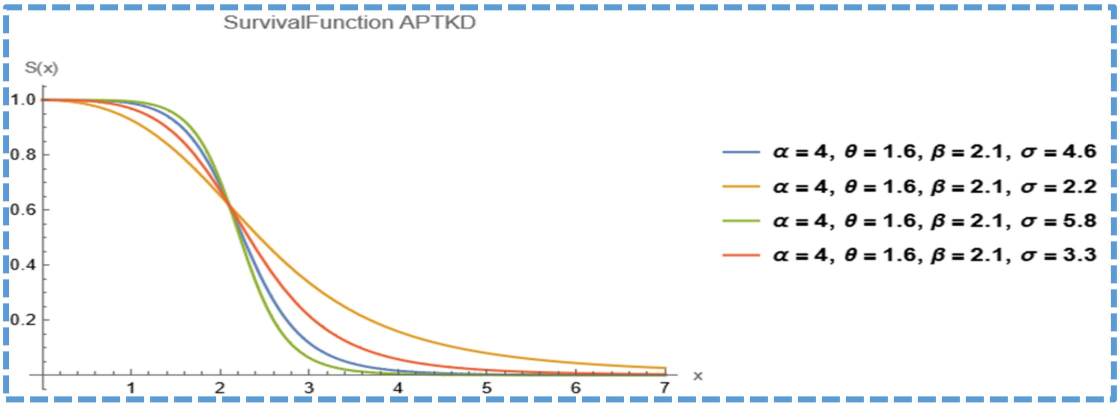



Figure 5 shows the behavior of the s(t)-distributed survival function Alpha Power Transformed kappa.

Alpha Power Transformed kappa of the risk function of a distribution.

$$h(x,\sigma,\theta\beta,\alpha)_{APTK} = \left[\frac{\frac{\sigma\theta}{\beta}\left(\frac{x}{\beta}\right)^{\theta-1}\left[\sigma+\left(\frac{x}{\beta}\right)^{\theta\sigma}\right]^{-\left(\frac{\sigma+1}{\sigma}\right)} \alpha^{\left[\frac{\left(\frac{x}{\beta}\right)^{\theta\sigma}}{\sigma+\left(\frac{x}{\beta}\right)^{\theta\sigma}}\right]^{\left(\frac{1}{\sigma}\right)}}}{1 - \frac{\alpha^{\left[\frac{\left(\frac{x}{\beta}\right)^{\theta\sigma}}{\sigma+\left(\frac{x}{\beta}\right)^{\theta\sigma}}\right]^{\left(\frac{1}{\sigma}\right)}} - 1}{\alpha - 1}}\right] \quad \dots (9)$$

The illustrative diagram in Figure (2-8) shows the behavior of the risk function for the distribution of ALTk

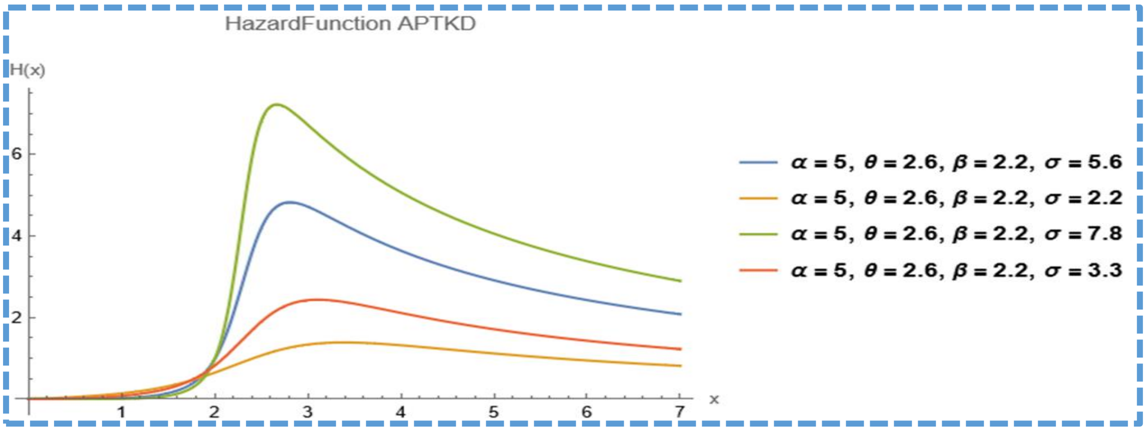



Figure 6 Behavior of the risk function h(t) of the Alpha Power Transformed kappa distribution.

**Maximum Likelihood Estimation Method**

The method of the greatest possibility (Maximum Likelihood Estimation) is one of the most common methods in the estimation process because it includes several good characteristics. The first to introduce this method is the researcher (Fisher in 1920), which aims to make the possibility function of random variables as large as possible. Assuming this The method is that the parameter to be estimated is a fixed value, meaning that the estimate will depend on the sample data for the observations, so we will need the Likelihood Function, the random variable used, and the parameters are estimated by equating the derivatives of the potential function with respect to the unknown parameters to be estimated with respect to zero.As these include The method has several characteristics, including efficiency and consistency, as well as having the least possible variance and stability (invariant), and it is more accurate than other estimation methods, especially when the sample size increases, and the possibility function can be defined mathematically.$L = f(x_1, x_2, \dots, x_n, \theta) = \prod_{i=1}^{n} f(x_1, \theta) f(x_2, \theta) f(x_3, \theta) \dots \; f(x_n, \theta)$ ... (18)

$$L = \prod_{i=1}^{n}\left[\left(\frac{\log\alpha}{\alpha - 1}\right)\frac{\sigma\theta}{\beta}\left(\frac{x_i}{\beta}\right)^{\theta-1}\left[\sigma\left(\frac{x_i}{\beta}\right)^{\theta\sigma}\right]^{-\left(\frac{\sigma+1}{\sigma}\right)}\alpha^{\left[\frac{\left(\frac{x_i}{\beta}\right)^{\theta\sigma}}{\sigma+\left(\frac{x_i}{\beta}\right)^{\theta\sigma}}\right]^{\left(\frac{1}{\sigma}\right)}}\right] \quad \dots (19)$$

Taking ln for both sides, we get the following equation

$$ln(L) = \left(\begin{array}{c} n\ln(\log\alpha) - n\ln(\alpha - 1) + n\ln(\sigma) + n\ln(\theta) - n\ln(\beta) \\ +(\theta - 1)\sum_{i=1}^{n}\ln\left(\frac{x_i}{\beta}\right) - \left(\frac{\sigma + 1}{\sigma}\right)\sum_{i=1}^{n}\ln\left[\sigma + \left(\frac{x_i}{\beta}\right)^{\theta\sigma}\right] + \ln(\alpha)\sum_{i=1}^{n}\left[\frac{\left(\frac{x_i}{\beta}\right)^{\theta\sigma}}{\sigma + \left(\frac{x_i}{\beta}\right)^{\theta\sigma}}\right]^{\left(\frac{1}{\sigma}\right)} \end{array}\right) \dots (20)$$

And by taking the derivative with respect to the parameters (β, θ, σ, α), and setting it equal to zero, we get the following equation:

$$\frac{\partial\, ln(L)}{\partial \alpha} = \frac{\sum_{i=1}^{n} \left(\frac{(\frac{xi}{\beta})^{\theta\sigma}}{(\frac{xi}{\beta})^{\theta\sigma} + \sigma}\right)^{\frac{1}{\sigma}}}{\alpha} + \frac{n}{\alpha Log[\alpha]} - \frac{n}{\alpha - 1} \quad \dots (21)$$

$$\frac{\partial \ln(L)}{\partial \theta} = \left( \frac{n}{\theta} + nLog\left[\frac{xi}{\beta}\right] + n\sigma Log[\frac{xi}{\beta} + \sigma] + \frac{\sum_{i=1}^{n} \left(\frac{(\frac{xi}{\beta})^{\theta\sigma}}{(\frac{xi}{\beta})^{\theta\sigma} + \sigma}\right)^{-1+\frac{1}{\sigma}} Log[\alpha]\left(-\frac{(\frac{xi}{\beta})^{2\theta\sigma}\sigma Log[\frac{xi}{\beta}]}{((\frac{xi}{\beta})^{\theta\sigma} + \sigma)^2} + \frac{(\frac{xi}{\beta})^{\theta\sigma}\sigma Log[\frac{xi}{\beta}]}{(\frac{xi}{\beta})^{\theta\sigma} + \sigma}\right)}{\sigma} \right) \quad \dots (22)$$

$$\frac{\partial \ln(L)}{\partial \sigma} = \left( -\frac{1}{\sigma} + \frac{n}{\sigma} + \frac{1+\sigma}{\sigma^2} + n\theta(\frac{\beta\sigma}{xi + \beta\sigma} + Log[\frac{xi}{\beta} + \sigma] + \frac{\sum_{i=1}^{n} (\frac{xi}{\beta})^{-\theta\sigma}\left(\frac{(\frac{xi}{\beta})^{\theta\sigma}}{(\frac{xi}{\beta})^{\theta\sigma} + \sigma}\right)^{1+\frac{1}{\sigma}} Log[\alpha](-\sigma + \theta\sigma^2 Log[\frac{xi}{\beta}] - ((\frac{xi}{\beta})^{\theta\sigma} + \sigma)Log[\frac{(\frac{xi}{\beta})^{\theta\sigma}}{(\frac{xi}{\beta})^{\theta\sigma} + \sigma}])}{\sigma^2} \right) \quad \dots (23)$$

$$\frac{\partial \ln(L)}{\partial \beta} = \left( -\frac{n}{\beta} - \frac{n(-1+\theta)}{\beta} - \frac{\sum_{i=1}^{n} xi\theta\sigma}{\beta^2\left(\frac{xi}{\beta} + \sigma\right)} + \frac{\sum_{i=1}^{n} \left(\frac{(\frac{xi}{\beta})^{\theta\sigma}}{(\frac{xi}{\beta})^{\theta\sigma} + \sigma}\right)^{-1+\frac{1}{\sigma}}\left(\frac{xi(\frac{xi}{\beta})^{-1+2\theta\sigma}\theta\sigma}{\beta^2((\frac{xi}{\beta})^{\theta\sigma} + \sigma)^2} - \frac{xi(\frac{xi}{\beta})^{-1+\theta\sigma}\theta\sigma}{\beta^2((\frac{xi}{\beta})^{\theta\sigma} + \sigma)}\right)Log[\alpha]}{\sigma} \right) \quad \dots (24)$$

Where we note that equations (24), (23), (22) and (21) are non-linear equations, so it is difficult to find them. Therefore, we will use one of the numerical methods to solve them, such as the Newton Ravsen method to estimate the parameters of the transformed distribution. We can get the estimated values (σˆ, θˆ,βˆ,αˆ) for the unknown parameters σ,θ,β,α, and then substitute the estimators (σˆ,θˆ,βˆ,αˆ) into the reliability function (8) we get the fuzzy reliability estimator for the method The greatest possibility.

The Simulation

Simulation is defined as an experimental mathematical method from imaginary reality that is used to solve complex problems in real reality. It is characterized by high flexibility because of the ability to re-experiment and test many times to reach accurate results to reduce time and cost.

It is considered one of the important stages, and it is a preliminary stage for the subsequent stages, and it is summarized in the following steps:

The stage of selecting the default values for the parameters of the proposed model (AP k), as shown in Table (3-1) below:

TABLE (1)Initial default values for the proposed parameters and models

| MODEL | α | σ | θ | β |
|---|---|---|---|---|
| Model 1 | 1.2 | 1.8 | 2.9 | **1.2** |
| Model 2 | 1.2 | 2 | 1.5 | **2.9** |

-1Choosing the sizes of the default sample, where four small, medium and large sizes were chosen (100, 75, 50, 25) in order to determine the effect of the sample size on the results of the estimate.

-2Repeat the experiment several times in order to get the best results.

At this stage, random data is generated that is suitable for the proposed model (Alpha Power kappa) by the inverse transformation method, and based on Equation No. (), which represents the quantitative function of the new expanded probabilistic model, as indicated in the second chapter within the theoretical aspect and according to the following steps:

In the beginning, random qi numbers are generated that follow the uniform distribution within the period{0,1}

q_i ~ UniformDistribution(0,1), i=1,2,…….,n

And that q_i represents a random variable that follows a regular distribution

Then we convert the data generated in the first step into the proposed distribution tracking data (Alpha Power kappa) using the inverse transformation method and according to equation (2-26), which represents the quantitative function of the new expanded probabilistic model, as indicated in the second chapter / the theoretical side, that is:

$$x = \beta\left(-\frac{\sigma\left(\frac{\mathrm{Log}[(-u+\frac{1}{1-\alpha})(1-\alpha)]}{\mathrm{Log}[\alpha]}\right)^{\sigma}}{-1+\left(\frac{\mathrm{Log}[(-u+\frac{1}{1-\alpha})(1-\alpha)]}{\mathrm{Log}[\alpha]}\right)^{\sigma}}\right)^{\frac{1}{\theta\sigma}}$$

After that, the parameters and the fuzzy reliability function are estimated for the proposed (Alpha Power kappa) distribution at this stage and for all the methods that have been explained in detail in the theoretical side, then a comparison is made between the estimations obtained for the parameters of the (Alpha Power kappa) distribution, by using the statistical mean squared error (MSE) criterion for the comparison between the estimations of the parameters and the fuzzy reliability function of the proposed (Alpha Power kappa) distribution.

Simulation results

After conducting the simulation process using a computer program, results were obtained for the estimations of the parameters and the fuzzy reliability function of the proposed model (APk) by the estimation methods used in the estimation process, as these results were classified into estimates in tables and graphs as will be shown later.

TABLE (2)

| N | Est. par. | MLE |
|---|---|---|
| **25** | $\alpha$ | 2.9904529 |
| | $\sigma$ | 1.8599455 |
| | $\theta$ | 2.9946507 |
| | $\beta$ | 0.2578375 |
| **50** | $\alpha$ | 3.0098428 |
| | $\sigma$ | 1.8146239 |
| | $\theta$ | 3.9979219 |
| | $\beta$ | 0.0848749 |
| **100** | $\alpha$ | 2.9997375 |
| | $\sigma$ | 1.8022253 |
| | $\theta$ | 2.9030961 |
| | $\beta$ | 1.2014422 |
| **150** | $\alpha$ | 2.9969046 |
| | $\sigma$ | 1.8248434 |
| | $\theta$ | 2.9356233 |
| | $\beta$ | 1.2163689 |

The following table No. (3) represents the values of the estimators of the experimental reliability function by the approved estimation methods for the (ti) values that were generated by simulation for the first model, with a comparison of the values of the sum of squares of error (mse) for the four sample sizes, as follows:

TABLE (3)

| n | ti | R-real | R-MLE | MSE |
|---|---|---|---|---|
| 25 | 0.72781 | 0.9514 | 0.92661 | **0.00061** |
| | 1.22686 | 0.84118 | 0.83093 | **0.00011** |
| | 1.26464 | 0.83076 | 0.82274 | **6.4E-05** |
| | 1.36513 | 0.80209 | 0.80047 | **2.6E-06** |
| | 1.46445 | 0.77266 | 0.7778 | **2.6E-05** |
| | 1.56564 | 0.7419 | 0.75413 | **0.00015** |
| | 1.56678 | 0.74155 | 0.75386 | **0.00015** |
| | 1.5993 | 0.73156 | 0.74615 | **0.00021** |
| | 1.88263 | 0.64417 | 0.67717 | **0.00109** |
| | 2.30996 | 0.52009 | 0.57018 | **0.00251** |
| | | | | |
| 50 | 0.88157 | 0.92376 | 0.87776 | **0.00212** |
| | 0.88257 | 0.92356 | 0.87753 | **0.00212** |
| | 0.88496 | 0.92308 | 0.87698 | **0.00213** |
| | 0.9981 | 0.89878 | 0.84996 | **0.00238** |
| | 1.09049 | 0.87676 | 0.82671 | **0.00251** |
| | 1.12554 | 0.86795 | 0.81764 | **0.00253** |
| | 1.13605 | 0.86525 | 0.8149 | **0.00254** |
| | 1.20783 | 0.84634 | 0.79293 | **0.00255** |
| | 1.21875 | 0.84338 | 0.76734 | **0.00255** |
| | 1.31244 | 0.81729 | 0.76674 | **0.00249** |
| n | | R-real | R-MLE | **MSE** |
| 100 | 0.75291 | 0.94732 | 0.94563 | **2.8E-06** |
| | 0.79339 | 0.94038 | 0.93882 | **2.4E-06** |
| | 0.7982 | 0.93953 | 0.93798 | **2.4E-06** |
| | 0.85722 | 0.92856 | 0.92723 | **1.7E-06** |
| | 0.91694 | 0.91653 | 0.91547 | **1.1E-06** |
| | 0.97825 | 0.90327 | 0.90251 | **5.8E-07** |
| | 1.01979 | 0.89378 | 0.89323 | **3E-07** |
| | 1.02643 | 0.89223 | 0.89172 | **2.7E-07** |
| | 1.07987 | 0.87939 | 0.87914 | **6.2E-08** |
| | 1.1096 | 0.87199 | 0.87188 | **1.1E-08** |
| n | | R-real | R-MLE | **MSE** |
| 150 | 0.58907 | 0.97078 | 0.97187 | **1.2E-06** |
| | 0.5999 | 0.96946 | 0.97065 | **1.4E-06** |
| | 0.65482 | 0.96226 | 0.96398 | **2.9E-06** |
| | 0.69156 | 0.95698 | 0.95909 | **4.5E-06** |
| | 0.73439 | 0.95034 | 0.95298 | **6.9E-06** |
| | 0.7943 | 0.94022 | 0.94366 | **1.2E-05** |
| | 0.90564 | 0.91887 | 0.92404 | **2.7E-05** |
| | 0.92669 | 0.91448 | 0.91999 | **3E-05** |
| | 0.97702 | 0.90355 | 0.90992 | **4.1E-05** |
| | 0.98708 | 0.90129 | 0.90784 | **4.3E-05** |

The application aspect

Data were collected representing the operating times until stopping or malfunctioning for a random sample consisting of (100) towers of a telecom company. Zain Iraq in the holy governorate of Karbala, for the year (2021) from January to August. When applying the real data in the proposed distribution (APk), we obtained estimated values for the parameters, as shown in the following Table (5):

Table No. (5) shows the estimated values of the parameters of the proposed distribution when applying the real data

TABLE (5)

| Distribution | $\sigma$ | θ | $\beta$ | $\alpha$ |
|---|---|---|---|---|
| **APTK** | 3.3932<br>(0.3932) | 2.2444<br>(0.0444) | 6.1931<br>(0.1931) | 0.2752<br>(0.0252) |
| **kappa** | 0.0011<br>(1.0903e-04) | 18.1722<br>(0.0016) | 0.000391<br>(10.172) | --------- |

In this research, the (Kolmogorov-Smirnov) test and (Chi Square) test were used to find out whether the real data collected by the researcher follows the proposed distribution or not according to the following two hypotheses.

H0: The data have) APTK) distribution.

H1: The data do not have) kappa) distribution.

The obtained test results are listed in the following tabl:

TABLE (6)

| **Distribution** | **Kolmogorov-Smirnov** | | **Chi Square test** | |
|---|---|---|---|---|
| | Statistic | P-Value | Statistic | **P-Value** |
| APTK | 0.0683 | 0.7132 | 8.5997 | **0.29324** |
| Kappa | 0.1846 | 0.0019 | — | |

It can be seen from Table (6) above that the P-value for the (Kolmogorov-Smirnov) test for the (APTK) distribution amounted to (0.7132), which is greater than the P-value for the same test for the (kappa) distribution, which amounted to (0.0019), and that the P-value - The value of the Chi Square test for the distribution of (APTK) amounted to (0.29324), and in both tests it is greater than (0.05) the level of significance, so the statistical decision is not to reject the null hypothesis Ho, that is, the real data follows our proposed distribution (APTK) in its distribution

The statistical criteria (AIC), (AICc) and (BIC) were adopted, which were explained in the second chapter within the theoretical aspect of the thesis, to see if the proposed distribution (APTK) is better in representing real data than the original kappa distribution. The results were obtained and listed in the following table

TABLE (7)

| Distribution | LL | | AIC | AICc | BIC | MSE |
|---|---|---|---|---|---|---|
| **APTK** | -207.8140 | | 423.6280 | 424.0403 | 434.0487 | 1.3182e-04 |
| **kappa** | -427.5973 | | 861.1946 | 861.4395 | 869.0101 | 2.463e-02 |

We note from Table (7) above that the proposed distribution (APTK) has obtained the lowest value for the three statistical criteria (AIC), (AICc) and (BIC) than the (kappa) distribution, according to which the (APTK) distribution is better in representing the real data used On the applied side of the message.
Table No. (8) below shows some statistical indicators of the proposed distribution when applying real data

TABLE (8)

| Coefficients | Value | Coefficients | Value |
|---|---|---|---|
| Mean | 4.5920 | Median | **4.5000** |
| Variance | 3.7436 | StandardDeviation | **1.9348** |
| Skewness | 0.024181 | Max | **8** |
| Kurtosis | 1.11276- | Min | **1** |

The following figure (7) represents the suitability of the proposed (APTK) distribution for real data compared to the original (kappa) distribution, as follows:

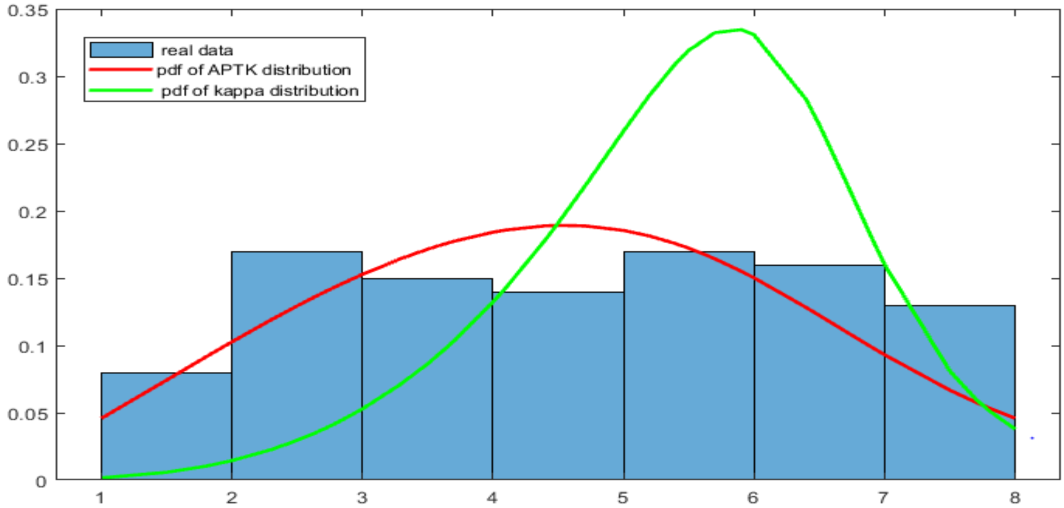



Figure 6

The following figure (8) shows the graph of the reliability function of the (APTK) distribution for the real data compared to the reliability function of the distribution itself for the estimated data.

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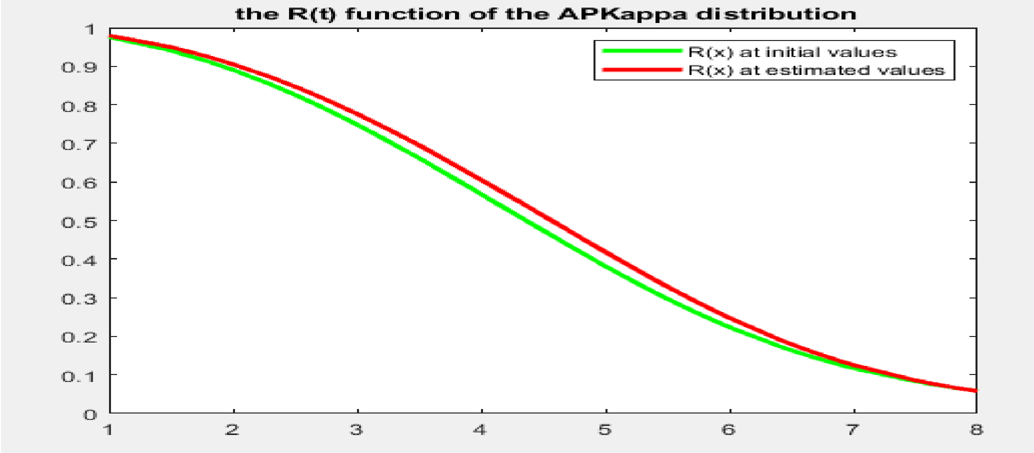


Figure 6

By applying the estimated parameters for each time from the real data collected by the researcher, we obtained the estimates of the fuzzy reliability function and the aggregate distribution function of the proposed distribution (APTK). (0.501735), meaning that we can count on the quality of these towers at a ratio of (0.501735)

## CONCLUSION

We conclude through practical application that the proposed alpha power kappa distribution is better than the original kappa distribution in the fit of the data to the applied side of the real data. We also conclude through simulation that the best method in estimating the parameters and estimating the fuzzy reliability function is the method of greatest possibility. "Considering the mean square error values, the maximum likelihood method provided good estimates."

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